FTD-HT-23-1137-68

AD 685028

FOREIGN TECHNOLOGY DIVISION

X-RAY DETERMINATION OF ELASTIC CONSTANTS E AND ν

by

M. M. Shved

Distribution of this document is unlimited. It may be released to the Clearinghouse, Department of Commerce, for sale to the general public.

EDITED TRANSLATION

X-RAY DETERMINATION OF ELASTIC CONSTANTS E AND ν

By: M. M. Shved

English pages: 9

Source: AN UkrSSR. Institut Mashinovedeniya i Avtomatiki. Voprosy Mekhaniki Real'nogo Tverdogo Tela (Academy of Sciences of the Ukrainian SSR. Institute of Machine Drive and Automation. Problems in the Mechanics of Real Solids), Izd-vo "Naukova Dumka", Kiev, 1964, pp. 128-135.

Translated by: J. Miller/TDBRO-2

DATA HANDLING PAGE

0'-ACCESSION NO.	TP9000071
98-DOCUMENT LOC	
39-TOPIC TAGS	x ray photography, poisson coefficient, elastic modulus
09-TITLE	X-RAY DETERMINATION OF ELASTIC CONSTANTS E AND ν
47-SUBJECT AREA	20, 12
42-AUTHOR/CO-AUTHORS	SHVED, M.M.
10-DATE OF INFO	-----64
43-SOURCE	AN UKRSSR. INSTITUT MASHINOVEDENIYA I AVTOMATIKI VOPROSY MEKHANIKI REAL'NOGO TVERDOGO TELA KIEV IZD-VO "NAUKOVA DUMKA" (RUSSIAN)
68-DOCUMENT NO.	FTD-HT-23-1137-68
69-PROJECT NO.	72301-78
63-SECURITY AND DOWNGRADING INFORMATION	UNCL, 0
64-CONTROL MARKINGS	NONE
97-HEADER CLASN	UNCL
76-REEL/FRAME NO.	1887 1916
77-SUPERSEDES	
78-CHANGES	
40-GEOGRAPHICAL AREA	UR
NO OF PAGES	9
CONTRACT NO.	
X REF ACC. NO.	65-BX6021568
PUBLISHING DATE	94-00
TYPE PRODUCT	TRANSLATION
REVISION FREQ	NONE
STEP NO.	02-UR/0000/64/000/000/0128/0135
ACCESSION NO.	

ABSTRACT

(U) The existing method of x-ray determination of elastic constants requires several tens of x-ray photographs to be taken, which is very tedious. In order to simplify the method of determining the elastic constants, author transforms the formulas of the relative strains in different directions for the case of a linear strained state in such a way that the stresses and modulus of elasticity are eliminated. Poisson's coefficient then is expressed in terms of three interplane distances of the atomic planes. As a result, for the determination of Poisson's coefficient, one may confine oneself to taking only three x-ray photographs. To find the modulus of elasticity one needs also to know the stresses arising in the object. This method was tested on specimens of armco-iron, A URS-501M apparatus was used. Author shows that a similar method also is possible for the plane stressed state, but that in this case at least four x-ray photographs must be taken.

AFSC FORM 4 AUG 66 (FTD OVERPRINT, DEC 68)

X-RAY DETERMINATION OF ELASTIC CONSTANTS E AND ν

M. M. Shved

We derive formulas for determining the elastic constants E and ν during uniaxial extension or compression in the plane-stressed state.

Experimentally determine the elastic constants of Armco iron during uniaxial extension.

It is known [2] that the calculation of residual stresses by the x-ray method using elastic constants obtained from mechanical tests leads to great discrepancies between the stresses measured by the x-ray method and mechanically-measured stresses. Therefore when measuring residual stresses by the x-ray method it is best to use elastic constants found directly from experiment.

At present there exists a method [1] which makes it possible, using x-ray photographs taken for various applied forces and at various angles with respect to the direction of the applied forces, to determine individually the elastic constants E and ν. However, the unwieldiness of the experiments makes this method unsuitable. For example, in order to determine the elastic constants for Armco iron it was necessary to take about 80 x-ray photographs at various stresses applied to the investigated specimen and at various angles to the direction of the applied forces [1]. In addition, the authors [1] presupposed that the elastic constants do not change with increasing stresses. In this paper we propose a simpler x-ray method for individual determination of E and ν.

Determining Elastic Constants E and ν with Uniaxial Extension or Compression

From the theory of elasticity [3], the deformation in direction ψ for uniaxial extension or compression (Fig. 1) will be

$$\varepsilon(\sigma_1, \psi) = \sigma_1\left[\frac{1+\nu}{E}\sin^2\psi - \frac{\nu}{E}\right], \tag{1}$$

where ψ is the angle between the normal to the applied stresses and the direction of the measured deformation; σ_1 is the magnitude of the applied stresses; ν is the Poisson ratio; E is Young's modulus.

When $\psi = 0$

$$\varepsilon(\sigma_1, \psi = 0) = -\sigma_1\frac{\nu}{E}. \tag{2}$$

If the relative deformation is represented as the deformation of the interplanar spacings, then from (1) and (2) we get, respectively,

$$\frac{d_\psi - d_0}{d_0} = \sigma_1\left[\frac{1+\nu}{E}\sin^2\psi - \frac{\nu}{E}\right]; \tag{3}$$

$$\frac{d_\perp - d_0}{d_0} = -\sigma_1\frac{\nu}{E}, \tag{4}$$

where d_ψ is the interplanar spacing of atomic planes in direction ψ for a specimen in the stressed state; $d_\perp$ is the interplanar spacing of atomic planes with the same indices in a direction normal to the applied forces for a specimen in the stressed state; d_0 is the interplanar spacing of atomic planes with the same indices for a specimen in the unstressed state.

If we subtract (4) from (3) we get

$$\frac{d_\psi - d_\perp}{d_0} = \sigma_1\frac{1+\nu}{E}\sin^2\psi. \tag{5}$$

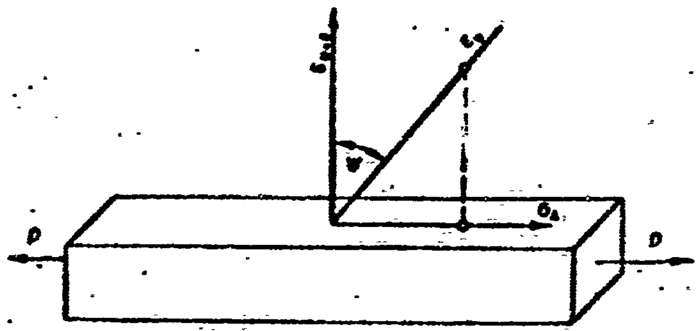

Fig. 1.

Dividing (4) by (5) we get

$$\frac{d_\perp - d_0}{d_\psi - d_\perp} = -\frac{\nu}{(1+\nu)\sin^2\psi}. \quad (6)$$

From this,

$$\nu = \frac{1}{\frac{d_\psi - d_\perp}{(d_0 - d_\perp)\sin^2\psi} - 1}. \quad (7)$$

From equation (7) it is obvious that we can determine the Poisson ratio by applying any forces under uniaxial extension or compression and measuring d_0, $d_\perp$, d_ψ and ψ. We need not know the magnitude of the applied forces in this case.

With uniaxial extension the interplanar spacings in the direction of the applied forces increase; in the direction normal to the applied forces they decrease. Obviously, there exists a direction along which the interplanar spacings do not change, i.e., the deformation in this direction equals zero. For such a direction $d_\psi = d_0$ and equation (7) assumes the form

$$\nu = \mathrm{tg}^2\psi_0. \quad (8)$$

An analogous expression can also be obtained for uniaxial compression. From equation (8) it is obvious that for uniaxial extension or compression the Poisson ratio is the square of the tangent of the angle between the normal to the applied forces and the direction along which deformation equals zero.

For an unstressed polycrystalline specimen the interplanar spacings of the atomic planes with identical indices are identical in all directions, and they can be represented in the form of a sphere of radius d_0. With uniaxial extension or compression the sphere assumes the form of an ellipsoid of rotation (axis of rotation – the direction of the applied forces). Connecting the points of intersection of the sphere and the ellipsoid of rotation with the center of the sphere we get the directions along which the deformation is zero ($d_\psi = d_0$). Figure 2 shows the intersection of the ellipsoid of rotation and the sphere for uniaxial extension (the cross section of the plane passing through the direction of the applied forces).

Thus, knowing the Poisson ratio, we can determine the direction along which deformation is zero, and vice versa. We should stress that the points of intersection of the sphere with the ellipsoid of rotation remain in place with a change in the applied forces until the Poisson ratio begins to depend on the magnitude of the deformation.

From (6) we get

$$d_\psi = d_\perp - \frac{1+\nu}{\nu}(d_\perp - d_0)\sin^2\psi. \tag{9}$$

From this it is evident that d_ψ is a linear function of $\sin^2\psi$, while the derivative

$$\frac{\partial d_\psi}{\partial \sin^2\psi} = -\frac{1+\nu}{\nu}(d_\perp - d_0) \tag{10}$$

characterizes the slope of line (9). At point $d_\psi = d_0$ this function will intersect the line d_0 = const. The projection of the point of intersection of these lines onto axis $\sin^2\psi$ makes it possible to determine $\sin^2\psi_0$ (Fig. 3). Knowing $\sin^2\psi_0$, we can determine ψ_0 and $tg^2\psi_0$ or ν. With a change in the stresses applied to the specimen, the point of intersection of these lines does not change (see Fig. 3), so long as the Poisson ratio remains constant, i.e., until the Poisson ratio begins to depend on the applied forces. From this it

is evident that the Poisson ratio can be determined for any stresses, which makes it possible to investigate its dependence on the magnitude of the applied stresses.

Having determined the Poisson ratio, we can easily determine Young's modulus, but for this we must know, besides $d_\perp$ and d_0, the magnitude of the stresses applied to the specimen, as follows from (4):

$$E = \frac{d_0 \sigma_1 \nu}{d_0 - d_\perp}. \tag{11}$$

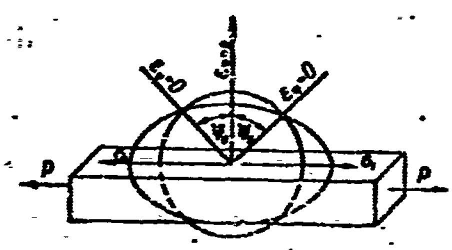

Fig. 2.

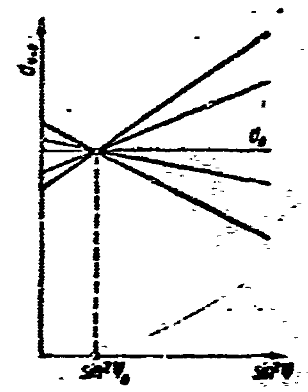

Fig. 3.

In this paper we have determined experimentally Young's modulus E and the Poisson ratio ν for three samples of Armco iron under uniaxial extension. The measurements were made on the URS-50IM installation in cobalt rays; calculation was done along line (310). The measurement and calculation results are given in the following table.

Sample No.	d_0, м	d_ψ, м	$d_\perp$, м	$\sin^2\psi$	σ_1, daN/mm^2	ν	E, daN/mm^2
1	$9.0475 \cdot 10^{-11}$	$9.0528 \cdot 10^{-11}$	$9.0450 \cdot 10^{-11}$	0.684	21.653	0.281	22050
2	$9.0470 \cdot 10^{-11}$	$9.0457 \cdot 10^{-11}$	$9.0460 \cdot 10^{-11}$	0.593	8.601	0.284	22100
3	$9.0473 \cdot 10^{-11}$	$9.0521 \cdot 10^{-11}$	$9.0453 \cdot 10^{-11}$	0.750	17.217	0.283	22080

It is also easy to calculate the elastic constants using equations (7) and (11) from the data in [1]; this can be done for each point of any of the graphs given in this work.

The accuracy of our proposed method of individual determination of the elastic constants coincides with that of [1], since in both cases the accuracy in determining E and ν depends on the accuracy of measuring the interplanar spacings, the accuracy of measuring ψ, and the value of the applied stresses.

Determination of the Elastic Constants in the Plane-Parallel State

From the theory of elasticity [2], for the plane-parallel state the deformation in direction ϕ, ψ

$$\varepsilon_{\varphi,\psi} = \frac{1+\nu}{E}(\sigma_1\cos^2\varphi + \sigma_2\sin^2\varphi)\sin^2\psi - \frac{\nu}{E}(\sigma_1+\sigma_2). \tag{12}$$

where ψ is the angle between the normal to the plane of the applied forces and the direction of the measured deformation; ϕ is the angle between the projection of the measured deformation onto plane σ_1, σ_2 and the principal stress σ_1; σ_1 and σ_2 are principal stresses; E is Young's modulus; and ν is the Poisson ratio.

For deformation in direction $\phi = \alpha$, ψ_1 (Fig. 4), from equation (12) we have

$$\varepsilon_{\alpha,\psi_1} = \frac{1+\nu}{E}(\sigma_1\cos^2\alpha + \sigma_2\sin^2\alpha)\sin^2\psi_1 - \frac{\nu}{E}(\sigma_1+\sigma_2); \tag{13}$$

for deformation in direction $\phi = \frac{\pi}{2} + \alpha$, ψ_2

$$\varepsilon_{\frac{\pi}{2}+\alpha,\psi_2} = \frac{1+\nu}{E}(\sigma_1\sin^2\alpha + \sigma_2\cos^2\alpha)\sin^2\psi_2 - \frac{\nu}{E}(\sigma_1+\sigma_2); \tag{14}$$

for deformation in a direction normal to the surface of the specimen (perpendicular to plane σ_1, σ_2)

$$\varepsilon_{\psi=0} = -\frac{\nu}{E}(\sigma_1+\sigma_2). \tag{15}$$

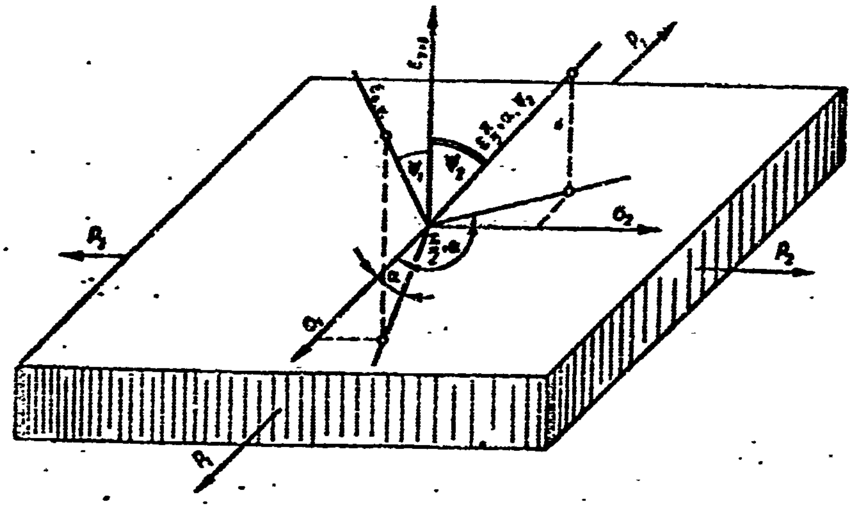

Fig. 4.

If deformation in each direction is represented as the deformation of the interplanar spacings, then from (13), (14), and (15) we get, respectively,

$$\frac{d_{\alpha,\psi_1}-d_0}{d_0}=\frac{1+\nu}{E}(\sigma_1\cos^2\alpha+\sigma_2\sin^2\alpha)\sin^2\psi_1-\frac{\nu}{E}(\sigma_1+\sigma_2); \quad (16)$$

$$\frac{d_{\frac{\pi}{2}+\alpha,\psi_2}-d_0}{d_0}=\frac{1+\nu}{E}(\sigma_1\sin^2\alpha+\sigma_2\cos^2\alpha)\sin^2\psi_2-\frac{\nu}{E}(\sigma_1+\sigma_2); \quad (17)$$

$$\frac{d_\perp-d_0}{d_0}=-\frac{\nu}{E}(\sigma_1+\sigma_2). \quad (18)$$

where d_{α,ψ_1} is the interplanar spacing of atomic planes in direction α, ψ_1 for a specimen in the stressed state; $d_{(\pi/2)+\alpha,\psi_2}$ is the interplanar spacing of atomic planes with the same indices in direction $(\pi/2)+\alpha$, ψ_2 for a specimen in the stressed state; $d_\perp$ is the interplanar spacing of atomic planes with the same indices in a direction normal to the surface of the specimen in the stressed state; d_0 is the interplanar spacing of atomic planes with the same indices for a specimen in the unstressed state.

If from (16) we subtract (18) and divide the resulting equation by $\sin^2\psi_1$, we get

$$\frac{d_{\alpha,\psi_1}-d_\perp}{d_0\sin^2\psi_1}=\frac{1+\nu}{E}(\sigma_1\cos^2\alpha+\sigma_2\sin^2\alpha). \quad (19)$$

Subtracting (18) from (17) and dividing the result by $\sin^2\psi_2$ we get

$$\frac{d_{\frac{\pi}{2}+\alpha,\psi_2}-d_{\perp}}{d_0\sin^2\psi_2}=\frac{1+\nu}{E}(\sigma_1\sin^2\alpha+\sigma_2\cos^2\alpha). \tag{20}$$

Let us add (19) and (20):

$$\frac{d_{\alpha,\psi_1}-d_{\perp}}{d_0\sin^2\psi_1}+\frac{d_{\frac{\pi}{2}+\alpha,\psi_2}-d_{\perp}}{d_0\sin^2\psi_2}=\frac{1+\nu}{E}(\sigma_1+\sigma_2). \tag{21}$$

Using (18), from (21) we find the Poisson ratio

$$\nu=\frac{d_0-d_{\perp}}{d_{\perp}-d_0+\frac{d_{\alpha,\psi_1}-d_{\perp}}{\sin^2\psi_1}+\frac{d_{\frac{\pi}{2}+\alpha,\psi_2}-d_{\perp}}{\sin^2\psi_2}}. \tag{22}$$

As can be seen from (22), in the asymmetric plane-parallel state we can determine the Poisson ratio from three x-ray photographs taken of the stressed specimen (perpendicular to plane σ_1, σ_2; at angle α, ψ_1; and at angle $(\pi/2)+\alpha$, ψ_2) and from one x-ray photograph taken of an unstressed specimen. It should be stressed that to determine the Poisson ratio in the plane-parallel state we need not know the magnitude nor the direction of the applied stresses.

For the directions along which deformation equals zero,

$$d_{\alpha,\psi_1}=d_0 \text{ and } d_{\frac{\pi}{2}+\alpha,\psi_2}=d_0;$$

then equation (22) has the form

$$\nu=\frac{1}{\frac{1}{\sin^2\psi_1^{(0)}}+\frac{1}{\sin^2\psi_2^{(0)}}-1}. \tag{23}$$

In the symmetric plane-parallel state

$$\nu=\frac{\sin^2\psi^{(0)}}{2-\sin^2\psi_0^{(0)}}. \tag{24}$$

From equations (23) and (24) it follows that in the case of the plane-parallel state the Poisson ratio, just as in the case of uniaxial extension or compression, is defined in terms of the angle between the normal to the applied forces and the direction along which deformation equals zero.

Knowing the Poisson ratio, we can, in the case of the symmetric plane-parallel state, determine from (24) the angle between the normal to the applied forces and the direction along which the deformation is zero, i.e.,

$$\psi^{(0)} = \pm \arcsin \sqrt{\frac{2\nu}{1+\nu}}. \tag{25}$$

Having determined the Poisson ratio, we can then from (18) determine Young's modulus (but for this we must know, besides d_0 and $d_\perp$, the sum of the principal directions), i.e.,

$$E = \frac{d_0 \nu (\sigma_1 + \sigma_2)}{d_0 - d_\perp}. \tag{26}$$

Thus, for the plane-parallel state we can determine the elastic constants from three x-ray photographs taken of the stressed specimen (at angle α, ψ_1; at angle $(\pi/2) + \alpha$, ψ_2; and in the direction normal to the plane of the principal directions), and one x-ray photograph taken of the unstressed specimen.

References

1. Macherauch, E., and P. Müller. Archiv für das Eisenhüttensessen, 29, No. 4, 1958.

2. Möller, H., J. Barbers. Mitteilungen aus dem Kaiser-Wilhelm Institut für Eisenforschung, 17, 1957, 1935.

3. Timoschenko, S., and J. Lessels. Festigkeitlehre, Berlin, 1928.

Received
15 June 1963